ALGEBRA II | Homework 0110 | Name:
DR. PAUL L. BAILEY | Monday, January 10, 2022

Because several students copied-without-understanding previous homeworks, homework will no longer be collected. Instead, it is your responsibility to complete and understand the homework, on your own, in preparation for the next exam. You may ask questions in class.

Definition 1. Let $p(x)$ be a polynomial, and let r be a real number.

The *multiplicity* of r as a zero of f is the largest nonzero integer n such that $(x-r)^n$ is a factor of $p(x)$.

Definition 2. Let $f(x) = \dfrac{p(x)}{q(x)}$, where p and q are polynomials. Let r be a real number.

We say that f has a *hole in the graph* (formally known as a *removable discontinuity*) at $x = r$ if $q(r) = 0$ and the multiplicity of r as a zero of p is greater than or equal to the multiplicity of r as a zero of q.

The *zeros* of f are the zeros of p which are not holes of f.

The *poles* of f are the zeros of q which are not holes of f.

Problem 1. Find the zeroes, holes, and poles of the following functions.

(a) $f(x) = \dfrac{x-1}{x-2}$

(b) $f(x) = \dfrac{x^2-1}{x^2-3x+2}$

(c) $f(x) = x^3 - 27x$

(d) $f(x) = \dfrac{1}{x-7}$

(e) $f(x) = \dfrac{x^3-7x+6}{x^2-1}$

Problem 2. Find the domain and range of the following functions.

(a) $f(x) = \dfrac{1}{x}$

(b) $f(x) = \dfrac{x-1}{x-2}$

(c) $f(x) = \dfrac{1}{x^2-6x-160}$

(d) $f(x) = \dfrac{x^2-16}{x^2-4}$

Definition 3. Given two function $f(x)$ and $g(x)$ whose codomain is $\mathbb{R}$, we define two new functions, the sum and product of these functions. The domain of the sum or product is the intersection of the domains. That is, for each $x \in \text{dom}(f) \cap \text{dom}(g)$, we define

- $(f+g)(x) = f(x) + g(x)$
- $(f \cdot g)(x) = f(x) \cdot g(x)$

Fact 1. The sum and product of polynomials are polynomials. The sum and product of rational functions are rational functions.

Definition 4. *Standard form* for a rational function f is $f(x) = \dfrac{p(x)}{q(x)}$, where p and q are written in standard form for a polynomial.

Problem 3. Write the following sums and products in standard form.

(a) Let $f(x) = 7x^3 + 3x - 7x + 8 + 2x^2$ and $g(x) = x^2 - 3x^3 + x - 15$. Find $(f+g)(x)$.

(b) Let $f(x) = x^2 - x + 5$ and $g(x) = x^2 + x + 10$. Find $(f \cdot g)(x)$.

(c) Let $f(x) = \dfrac{x-1}{x-2}$ and $g(x) = \dfrac{x-3}{x-4}$. Find $(f \cdot g)(x)$. Find $\text{dom}(f \cdot g)$.

(d) Let $f(x) = \dfrac{x-1}{x-2}$ and $g(x) = \dfrac{x-3}{x-4}$. Find $(f+g)(x)$. Find $\text{dom}(f \cdot g)$.

(e) Let $f(x) = \dfrac{x^2-1}{x+2}$ and $g(x) = \dfrac{x+2}{x^2-1}$. Sketch the graph of $(f \cdot g)(x)$.